Performance Comparison of Interactive Multiple Model - Extended Kalman Filter for Bearings - only Tracking

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Abstract– **Online tracking of maneuvering target is a highly non-linear and challenging problem in which the unknown state of the target is estimated from noisy observations. The bearings-only tracking has the advantage of direct measurement of the target location, from the beam crossing. To process the non-linear measurements, Extended Kalman Filter(EKF) and Unscented Kalman filter(UKF) are generally used. If the target is maneuvering and switching among different models like constant velocity (CV), constant acceleration (CA) or constant turn (CT) interactive multiple models (IMM) are employed. In this paper, IMM-EKF is comprehensively evaluated and its performance is compared with conventional KF and EKF in terms of robustness, computational complexity and error performance.**

Keyword: Bearings only tracking, Extended Kalman filter, Interactive Multiple Model, Noise Covariance.

1. Introduction

The goal of the target tracking system is to follow the target trajectory using useful information about the target's state from the sensor observations. The bearing-only tracking is of interest for passive tracking of moving targets. It involves the determination of the trajectory of a target based on the non-linear measurements, the bearing to the multiple sensors. In many tracking applications, a Kalman filter is used to estimate the position, velocity, and acceleration of a maneuvering target from noisy measurements at high data rates [8-11]. In case of bearings measurement, the dynamic equations turn out to be nonlinear which lead to the application of the nonlinear models like extended Kalman filters to tracking problems. The dynamic equations are linearized on the state at each instant resulting in a system of equations with time-dependent coefficients. Further considerations regarding the non-linear approach can be found in [1, 2]. In the case of maneuvering targets, where the target switches between multiple states, the interactive multiple model (IMM) is employed. IMM uses a bank of filters processing in parallel [3], with each filter acting on a dynamic model. The robustness of IMM Kalman filter is not completely discussed in the literature. This paper provides

an extensive comparison of filters using six different performance parameters.

The paper organization is as follows. First filtering Algorithms are briefly discussed. In Section 2 Mathematical modeling of target tracking, in Section 3 Simulations and results and in section 4 Conclusions and future scopes are discussed.

1.1. Kalman Filter

Kalman filter [4, 5, 6, 7] is a recursive filter which estimates the state of a dynamic system from the noisy measurements. Only the estimated state from the previous time step and the current measurement are needed to compute the estimate of the current state. It has two distinct phases (Fig. 1.):

> *Predict:* The time update or predict state projects the current state estimate ahead of time.
> *Correct:* The projected estimates are adjusted by actual measurements at that time.

The time update phase and the measurement update phase consists of equations shown in fig (1) [4, 6, 8, 9, 10]:

1.2. Extended Kalman Filter

EKF is an non-linear version of kalman filter which linearizes about the current mean and covariance. The state transition and observation models are non-linear functions or differentiable functions of the state.

The state of the target evolves according to the equation [11, 12]

$$X(k) = f(X(k-1), k-1) + Gw_k. \tag{1}$$

The measurement is given by

$$Z(x) = h(X(k), k) + v_k. \tag{2}$$

The functions f, h are non-linear functions of X, they cannot be applied to covariance directly. The matrices of partial derivatives of A, H are computed. The equations present in the EKF algorithm [4, 5, 6] are shown in Fig. 2. Q_{K-1} is the co-variance of the process noise while R_k is the co-variance of the measurement noise. The predicted state $X(k)'$ and the associated state prediction covariance $P(k)$ of the original non-linear model are linearly

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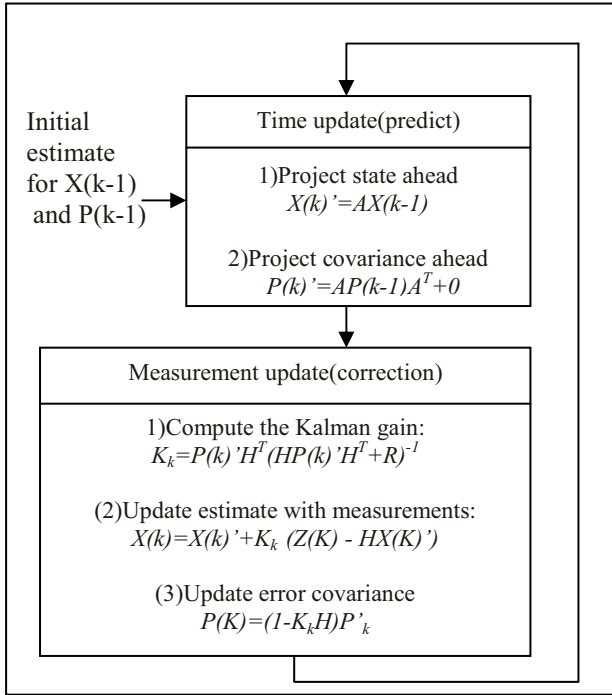Email addresses: ravikumar@nitw.ac.in, Ph: +91 9490853570

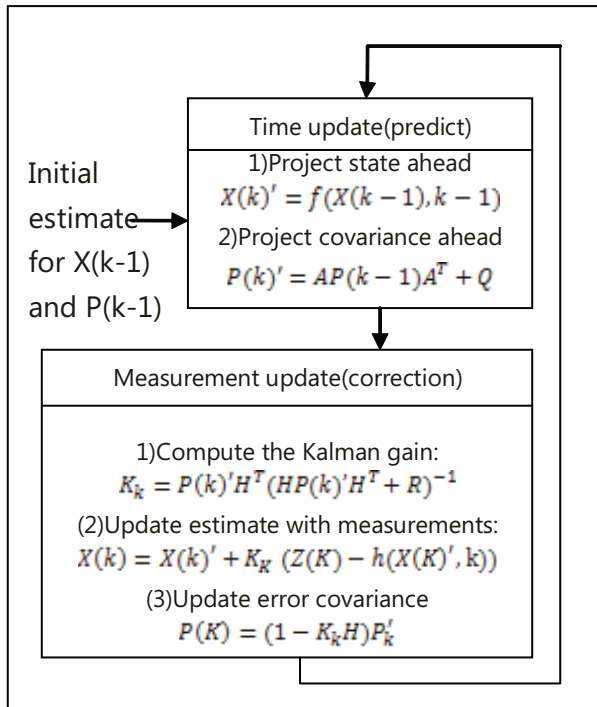**Fig. 1.** Steps in Kalman filter.

Fig. 2. Steps in EKF algorithm.

approximated by EKF. To obtain $X(k)$, the nonlinear function in Equation (1) is expanded in vector Taylor series around the last state estimate $X(k-1)$[12].

$$f(x) = f(\hat{x}) + \bar{\nabla}_x f(\hat{x})(x - \hat{x}) + \frac{1}{2}\bar{\nabla}_x^2 f(\hat{x})(x - \hat{x})^2 + H.0.T. \quad (3)$$

Truncating the terms up to first and second order yield the first and second order Extended Kalman Filter respectively.

1.3. IMM-EKF

Imm [3, 12, 13] is a versatile tool for adaptive state estimation in systems whose behavior pattern changes with time. It is a hybrid filter system comprised of a finite number of system models, used to solve the model behavior problem. A Markov transition matrix is specified to give the probability that the target is in one of the models of operation. At each new measurement the model probabilities are updated and the state matrix $X(k)$ is calculated using the resultant weighing factors. The scheme of IMM is shown in Fig. 3.

One cycle of IMM consists of five major steps [3]:

Step 1: Calculation of mixing probabilities

$$\mu_{i/j}\left(\frac{k-1}{k-1}\right) = P\left\{\frac{M_i(k-1)}{M_j(k)}, z^{k-1}\right\}$$

$$= \frac{1}{\bar{c}_j}p_{ij}\mu_j(k-1) \quad (4)$$

Where $\bar{c}_j = \sum_{i=1}^r p_{ij}\mu_j(k-1)$ is the predicted mode probabilities and r different models.

Step 2: Calculate the mixed initial condition

$$\hat{x}^{ri}\left(\frac{k-1}{k-1}\right) = \sum_{i=1}^r \hat{x}^i\left(\frac{k-1}{k-1}\right) * \mu_{i/j}\left(\frac{k-1}{k-1}\right) \quad (5)$$

$$P'_j\left(\frac{k-1}{k-1}\right) = \sum_{i=1}^r \mu_{i/j}\left(\frac{k-1}{k-1}\right)$$
$$* \left\{ \begin{array}{c} P^i\left(\frac{k-1}{k-1}\right) + \left[\hat{x}^i\left(\frac{k-1}{k-1}\right) - \hat{x}^{ri}\left(\frac{k-1}{k-1}\right)\right] \cdot \\ \left[\hat{x}^i\left(\frac{k-1}{k-1}\right) - \hat{x}^{ri}\left(\frac{k-1}{k-1}\right)\right]^T \end{array} \right\} \quad (6)$$

Step 3: Perform mode-matched filtering and calculate the likelihood function corresponding to the filter

The estimate $\hat{x}^{ri}\left(\frac{k-1}{k-1}\right)$ and corresponding covariance $P'^j\left(\frac{k-1}{k-1}\right)$ are used as inputs to the filter matched to $M_j(k)$ which uses the measurements $z(k)$ to yield $\hat{x}^j(\frac{k}{k})$ and $P^j(\frac{k}{k})$.

The filter equations are shown in Fig. 2

Step 4: Update model probability

$$\mu_j(k) = \frac{1}{c} \wedge_j(k)\bar{c}_j \text{ where } c = \sum_{j=1}^r \wedge_j(k)\bar{c}_j \quad (7)$$

Step 5: combine model-conditioned estimates and covariance.

$$\hat{x}(\frac{k}{k}) = \sum_{i=1}^r \hat{x}^i(\frac{k}{k})\mu_i(k) \quad (8)$$

$$P(\frac{k}{k}) = \sum_{i=1}^r \mu_i(k)\left\{ P^i(\frac{k}{k}) + \left[\hat{x}^i(\frac{k}{k}) - \hat{x}(\frac{k}{k})\right]\left[\hat{x}^i(\frac{k}{k}) - \hat{x}(\frac{k}{k})\right]^T \right\} \quad (9)$$

2. Problem Description

The target motion can be described by a large number of models. It can move with constant velocity, constant acceleration or constant turn. Here we consider a CV-CT-CV-CT-CV-CT-CV model.
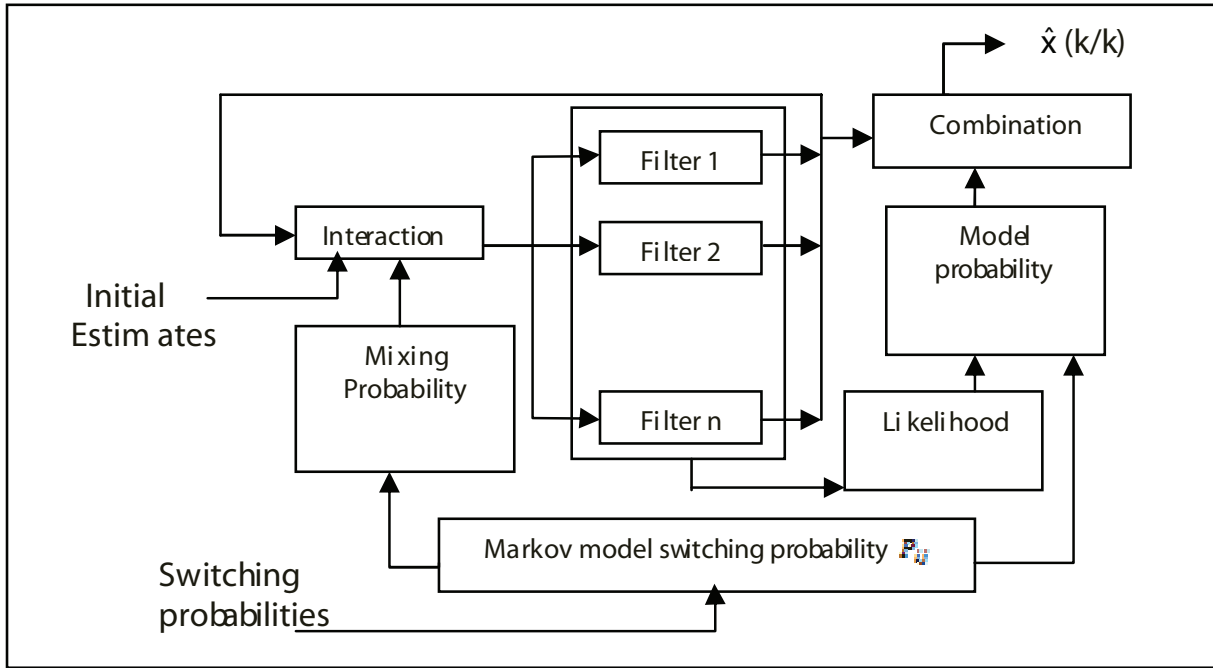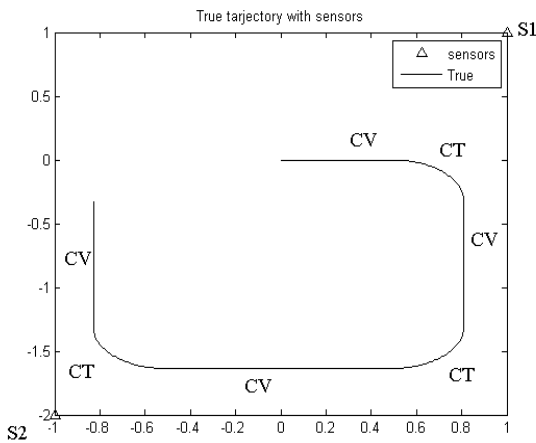
Fig. 3. Interacting Multiple Model.

Fig. 4. True geometry of target.

2.1. *Constant velocity(CV) model [14]*

The state vector of the target is given by $X(k) = [x_k, y_j, v_{xk}, v_{yk}]^T$.

Where T denotes the matrix transpose. The state equation for the target motion could be approximated with a linear equation of the form

$$X_{k+1} = A_k x_k + G w_k.$$

Where x_k is the state vector that contains state variables at time k, and $w_k \sim N(0, Q_k)$ which is assumed as zero mean white noise with covariance Q_k (called process noise). The transition matrix A and the process noise covariance matrix ($Q_k = E[w_k w_k^T]$) are given by

$$A = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Q = q_1 \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ 0 & 0 & \frac{T^3}{3} & 0 \\ 0 & 0 & 0 & \frac{T^3}{3} \end{bmatrix}$$

Where q_1 is the level of power spectral density.

2.2. *Constant turn(CT) model [14]*

The constant turn model has an angular velocity component along with the position and velocity components. The state vector is of the form $X(k) = [x_k, y_k, v_{xk}, v_{yk}, w_k]$. The transition matrix is given by

$$A = \begin{bmatrix} 1 & 0 & \sin(wt)/w & (\cos(wt) - 1)/w & 0 \\ 0 & 1 & (\cos(wt) - 1)/w & \sin(wt)/w & 0 \\ 0 & 0 & \cos(wt) & -\sin(wt) & 0 \\ 0 & 0 & \sin(wt) & \cos(wt) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $wt = w * T$, the process noise co-variance matrix is calculated as $Q_k = E[w_k w_k^T]$

3. Simulations and Results

3.1. *Path description*

The target is considered to fly in the (x, y) plane with an initial position [0, 0] and an initial velocity [1 m/s, 0 m/s]. Measurements are taken for every 0.01 sec (T). Fig. 4 shows the 7-motion sequence (CV-CT- CV-CT- CV-CT-CV):

(1) CV for the first 0.5 seconds(50 time intervals) (2) CT for the next 0.5 sec s(51–100 time intervals) (3)CV for the next 1 sec (time intervals 101–200) (4)CT for the next 0.5secs (intervals 201–250) (5)CV for the next 1sec (time intervals 251–350) (6)CT for the next 0.5secs (intervals 351–400) (7)CV for the next 1sec (time intervals 401–500)

3.2. Measurements

Two sensors are assumed to be at (i)S1:[1,1]; (ii)S2:[−1,−2]. The bearings from these two sensors to the target are calculated for every T sec.

$$[Z] = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} sd * randn \\ sd * randn \end{bmatrix} \text{ where } \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \tan^{-1} \frac{(y_k - S_1(2))}{(x_k - S_1(1))} \\ \tan^{-1} \frac{(y_k - S_2(2))}{(x_k - S_2(1))} \end{bmatrix} \quad (10)$$

Where *sd(=0.01)* is the covariance of measurement noise.

3.3. Application of filters

As the measurements are non-linear, the Kalman filter cannot be directly applied to the readings. We have two equations (10) and two unknowns (x_k, y_k), the coordinates of the target are calculated by solving the two equations.

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} \frac{(S_2 - S_1(2)) + (S_1(1)*\tan(z(1)) - S_2(1)*\tan(z(2)))}{(\tan(z(1)) - \tan(z(2)))} \\ \frac{\{S_1(2)*\tan(z(2)) - S_2(2)*\tan(z(1)) + (S_2(1) - S_1(1))*\tan(z(2))*\tan(z(1))\}}{(\tan(z(1)) - \tan(z(2)))} \end{bmatrix} \quad (11)$$

The measurement matrix is given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The EKF and IMM-EKF are non-linear filters and can be applied directly on the bearing measurements. For EKF the measurement matrix *h* is a function which takes X_k and $S = [S_1, S_2]$ as inputs and calculates Z_k.

H is the Jacobian matrix of *h*, which is obtained by calculating the partial derivatives of the terms of *h*

$$H_k = \nabla_{xk} h \quad (12)$$

For IMM-EKF, the initial model probabilities $\mu = [\mu_1 \ \mu_2]^T$ corresponding to the maneuvering and non-maneuvering mode are taken as 0.95 and 0.05 respectively. The Markov transition matrix P_{ij} which specifies the switching from mode *I* to mode *j* is a design parameter. It is taken as

$$P_{ij} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

The values in are taken on the basis that, in the initial stage, the target is more likely to be in non-maneuvering mode and its probability to switch to maneuvering mode (P_{12}) is low.

3.4. Comparison of filter outputs

The performance of the filters is compared by computing and plotting the following parameters:

1. Measured and estimated *x, y* positions
2. Mean in *x, y* position errors. Its specifies the average error between the true value and the measurements.

$$\text{Mean in x position error} = \frac{1}{N}\sum_{i=0}^{N}(x_k - x'_k) \quad (13)$$

Similarly for y position. Where x is the measured position and *x'* is the estimated position.
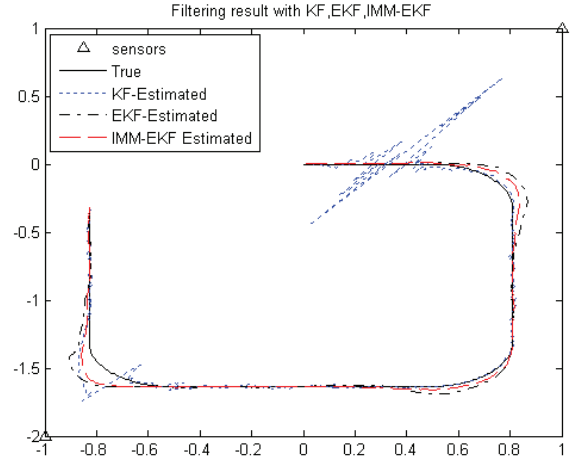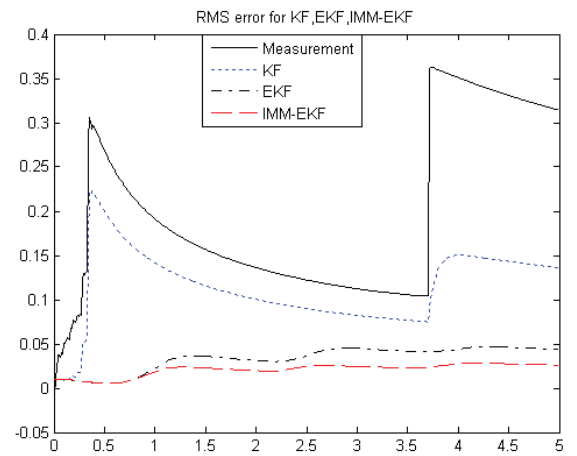

Fig. 5. Outputs from 3filters for *sd* = 0.01.

Fig. 6. RMS error in position with sd=0.01.

3. The percentage fit error(PFE) in x & y positions:

$$\text{PFEx} = 100 * \frac{norm(x - x')}{norm(x)} \quad (14)$$

Similar expression for y

4. Root mean square position error:

$$\text{RMSPEx} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_k - x'_k)^2}, \quad (15)$$

similarly RMSPE is calculated in y direction.

5. Root sum square position error:

$$\text{RSSPEx} = \sqrt{\sum_{i=1}^{N}(x_k - x'_k)^2} \quad (16)$$

6. The root sum variance

$$\text{RSvarP} = \sqrt{P_x + P_y} \quad (17)$$

where P_x and P_y are the diagonal elements in P corresponding to positions x & y.

From the figures 6–9 it can be seen that the RMS and RSS errors in the position and velocity estimations is least in

Table 1. Comparison of filters

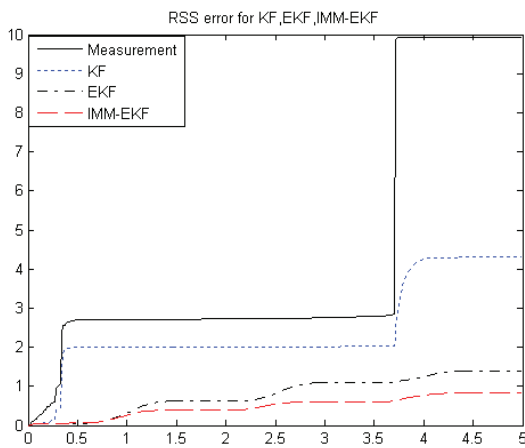|		Direct Measurement	KF	EKF	IMM-EKF
Mean error (in mts)	X	0.6574	0.4456	0.1612	0.1327
	Y	1.0132	0.3926	0.1987	0.1670
PFE	X	7.0921	1.4990	0.2780	0.2268
	Y	6.9583	0.9866	0.2348	0.1890
RMSPE-P (in mts)		6.5345	1.0639	0.2303	0.1862
RSSPE-P (in mts)		206.6385	33.6445	7.2826	5.8874
RMSPE-V (in mts)		-	1.6111	0.5611	0.5029
RSSPE-V (in mts)		-	50.946	17.743	15.902
PSvarP		-	0.0527	0.0582	0.0611

Fig. 7. RSS error in position with sd=0.01
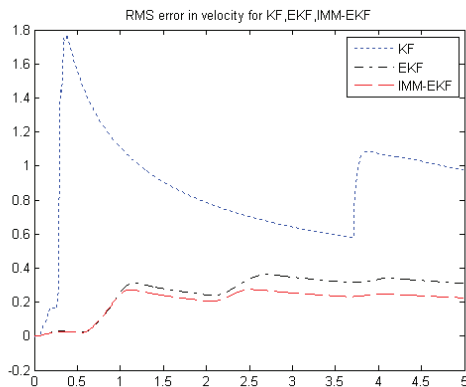

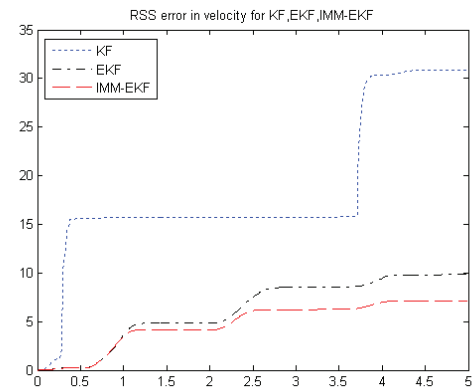**Fig. 9.** RMS error in velocity with sd=0.01.

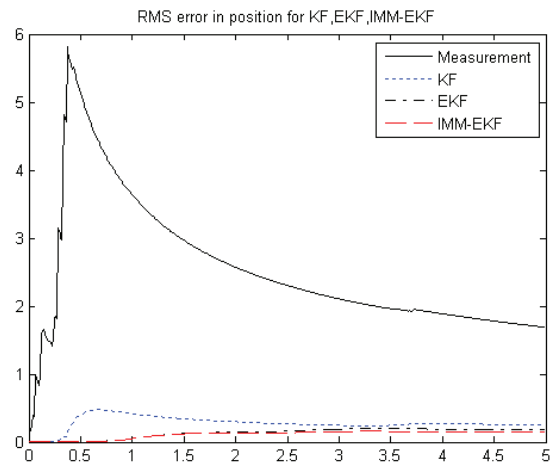**Fig. 8.** RMS error in velocity with sd=0.01.

Fig. 10. RMS error in position with sd=0.1.

IMM-EKF compared to the KF and EKF for a standard deviation of 0.01.

7. The time complexity for the three filters is calculated. (CC=computation complexity)

	KF	EKF	IMM-EKF
CC(in secs)	0.000632	0.000718	0.00147

Similar simulations are carried out for standard deviation of sd=0.1.

From the figures 10–13 it can be seen that the RMS and RSS errors in the position and velocity estimations is least in IMM-EKF compared to the KF and EKF for a standard deviation of 0.1.

4. Conclusions and Future Scope

From the above simulations and results we can conclude that IMM-EKF performs better compared to KF and EKF for bearings only tracking. In this paper robustness of IMM-EKF is proved in worst conditions also. The performance is evaluated by considering noise of standard deviations, $\sigma = 0.01$ and $\sigma = 0.1$ and mean error, PFE in X and Y directions, RMS, RSS for position and velocity are calculated, in all of which IMM-EKF has performed better. Computational complexity is comparatively high but that problem can be overcome by implementing the algorithm
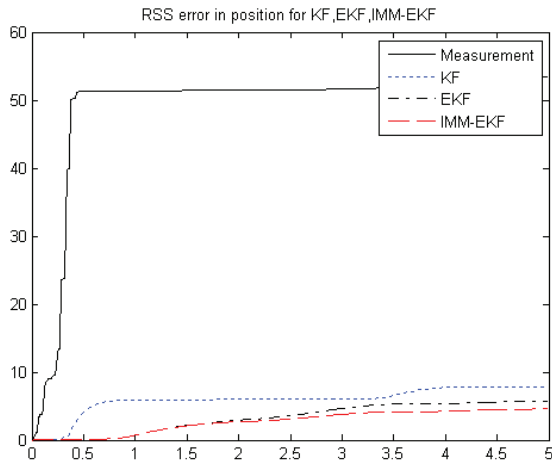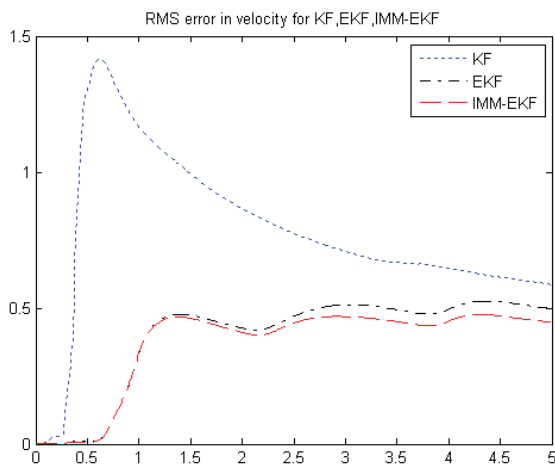
Fig. 11. RSS error in position with sd=0.1

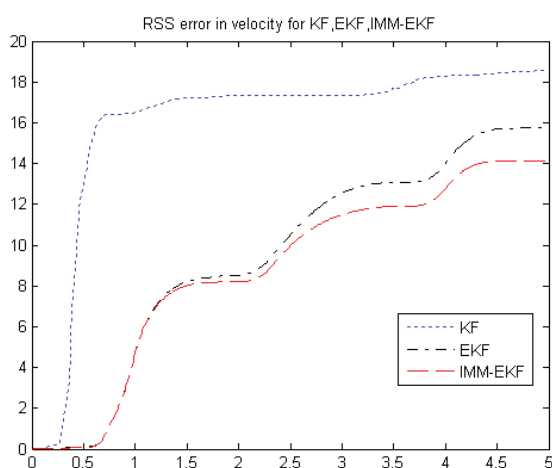**Fig. 12.** RMS error in velocity with sd=0.01.

Fig. 13. RSS error in velocity with sd=0.1.

on a high speed DSP processor. In future, real time implementation of IMM_UKF on DSP processor or FPGA can be carried and comparison of IMM-EKF with IMM-UKF can be done.

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